PHILOSOPHICAL TRANSACTIONS.

I. *An Account of the Construction of a Fluid Lens Refracting Telescope of eight inches aperture, and eight feet and three quarters in length, made for the Royal Society by* George Dollond, *Esq. F.R.S. By* Peter Barlow, *Esq. F.R.S. F.R.A.S. M.C.P.S. Cor. Mem. Inst. France, &c.*

Read November 22, 1832.

In my former papers on the construction and performance of fluid refracting telescopes with open lenses, I have pointed out the great variety of cases included in the general formula, which have since been increased by the ingenious construction of Mr. Rogers, and have referred to the difficulty of carrying one's mind through all their intricacy, so as to select, independently of experiment, from amongst the several cases, that which would produce the best result. The form I gave to my original construction was founded principally upon the idea of lengthening the focus beyond the length of the tube, and as far as that object was concerned, the result was perfectly satisfactory; but, as I have stated in my description of that instrument, it was found, as this principle was extended, that the perfect part of the field became more contracted, so as to render it questionable at what point to stop, to produce upon the whole the best effect. Other considerations also presented themselves, and I wished therefore to have the means of making certain preliminary experiments, with a view to the determination of a few such practical points, before selecting out of the multiplicity of arrangements (all theoretically true,) that which should be adopted in the construction of so large a telescope as that which I ventured to propose in my last paper, Phil. Trans. for 1831. The advantage, if not the

The Royal Society is collaborating with JSTOR to digitize, preserve, and extend access to *Philosophical Transactions of the Royal Society of London.* www.jstor.org

absolute necessity, of such experiments in a novel case of this kind must be obvious to every one who has had any experience in practical optics, where having several objects to fulfil, interfering more or less with each other, it is only by experiment it can be ascertained how far an advantage can be pushed on one side without making too great a sacrifice on another; and such experiments are too expensive to be conveniently carried on by a private individual who has no other object in their success than the advancement of optical and astronomical science.

The Council of the Royal Society therefore appointed a committee to report upon my former paper, who were of opinion that it would be advisable to proceed at once to the construction of an eight-inch telescope for the Royal Society, the success of which might decide the question of proceeding upon the scale I had proposed. They accordingly ordered a telescope of the above dimensions of Mr. Dollond, leaving to me the arrangements relative to curvatures, focal length, &c. After due consideration, I determined on the adoption of a form and principle of construction which I thought likely to embrace the most advantages, and I am in hopes the result will not be considered unsatisfactory, although I have in the progress of the construction seen one or two changes which, in a future case, might be advantageously adopted.

The aperture of the telescope is eight inches clear glass, and focal length eight feet and three quarters. The spherical aberration is perfectly corrected, the field is open and flat, with abundance of light,—all very desirable qualities in an astronomical telescope; with respect to colour, there is perhaps more outstanding on the violet side of the spectrum, than is generally found in the usual refractor, particularly towards the limits of the field. I estimate the extent of the field by the product of the minutes it contains, multiplied by the power, and I find this product with my four powers, varying from 100 to 450, ranges between 3000 and 3450, which rather exceeds perhaps the usual fields of refractors; at all events there is a great improvement in this respect as compared with my former instrument.

As to the length, although I am unwilling to think that upon the whole the telescope is too short, yet it must be admitted that where minute definition is wanted, an advantage is gained by contracting the aperture, but the whole is certainly available for bringing to light very small stars; and probably more distinctness with the whole aperture might be derived from a little increase in the aperture of the fluid lens.

Having made these few general remarks, I shall endeavour to explain the reasons which induced me to select the particular case I have done, and the principles on which I have established the equations of condition. In my former paper I have explained my reason for using a double instead of a single front lens. It moreover occurred to me, that although the chromatic dispersion of a lens of any medium, or the coloured part of its focus, bears a fixed ratio to its mean focal length, this is not the case at the first immersion of a ray and during its progress in the medium itself, a remark I do not remember to have seen made by any optical writer. Also, as the dispersion of a lens depends upon the relative indices of the extreme and mean rays of the spectrum, this will vary according to the medium from which a lens receives and into which it transmits those rays, so that the dispersion of the fluid will be much greater receiving and transmitting rays into glass than into air.

To take advantage of this circumstance I resolved to make all the correction for colour and aberration in the passage of the rays from one fluid shell to the other, by causing the mean rays from the front compound lens to impinge perpendicularly on the first surface of the fluid shell, and again perpendicularly on the fourth surface, and thereby, of course, preventing any further chromatic development.

By this arrangement, the dispersion or coloured part of the focus of the front compound lens is reduced on entering the first shell of the fluid lens, in the ratio of about 3 to 2, while that of the fluid is so much increased, that the dispersive ratio of the two during the passage of the ray through them is reduced to ·08, whereas, in the usual refractor, it is about ·60 or ·65; the dispersive ratio is therefore only about one eighth of its usual amount, and the spherical aberration is also reduced by the nature of the construction in about the same proportion, that is, to about one third, in consequence of the double front lens, and to about one third again by the focus being nearly three times as long as that of the crown lens of the usual refractor.

There was only one doubtful point connected with this form, which was, that as in providing for the correction of the spherical aberration we can only effect this for one index (generally the mean), the aberration of the extreme rays must remain imperfectly corrected, which imperfection is greater as the indices of the extreme rays bear a higher ratio to the mean index; and thus it constantly happens in constructions of this kind, that while we are advancing towards perfection on one hand, a corresponding evil enters on the other, a circumstance which renders preliminary experiments so highly necessary in everything connected with practical optics. As however the whole linear amount of the aberration of the front lens does not amount to $\frac{7}{100}$ths of an inch, I fully determined upon adopting this principle of construction, as it appeared to me that, with so little to correct, the imperfection must necessarily be very inconsiderable. This circumstance, however, led me to consider what might be the advantages of making the correction for the spherical aberration of the front lens, by means of the glass shells of the fluid lens, reducing that of the fluid either to zero or to a minimum. And on making independent calculations on both hypotheses, I found, what was perfectly unexpected to me, that the same fluid lens would answer both cases by simply inverting it, or so nearly so, at least, as not to render it necessary to have a second fluid lens made to decide the question.

With these decided advantages, according to either principle, it was obvious that I might venture upon a much shorter telescope than would be required for an eight-inch aperture of the usual kind of refractor; but the want of preliminary experiments made it impossible to know to what extent this shortening might be carried. I determined therefore on eight-feet focus; but the nearest I could come to this length without having new tools made, was eight feet nine inches.

Another principle, which was thought desirable, was to divide the refraction pretty equally between all the four surfaces of the two front lenses; and this condition was quite consistent with the adoption of two equal convex lenses, as will be seen immediately by taking ·500 as an approximate index of crown glass, and ·100 inches as the compound focus; for then the first refraction will make the rays converge to 300 inches, the second refraction to 200 inches, the third to 150 inches, and the fourth to 100 inches. If however I had to repeat the experiment, I should prefer making all the refractions at the first three surfaces, by causing the light to fall perpendicularly on the last, as in that case the passage of the rays, after entering the second lens, to the focus, would (employing the first of the above principles,) be made in precisely the same way as in an uniform cylinder of glass, as in Mr. CODDINGTON'S microscope; with the exception of the slight refraction they would sustain in passing through the fluid, which must necessarily be very inconsiderable, as its negative focal length is more than six hundred inches. Moreover, besides this advantage, which may be perhaps rather apparent than real, this form would have given less spherical aberration, and consequently the curvature of the fluid might have been still further reduced, which is at all events a real advantage, but it did not occur to me till I had proceeded too far to make any alteration.

It will be seen from what has now been stated, that the conditions I proposed to myself were:

1st, That the front lens should be composed of two plano-convex lenses of equal focal length.

2ndly, That the curvatures of the fluid shells should be such that the mean rays, after leaving the front lens, should fall perpendicularly on the first surface.

3rdly, That the corrections for colour and aberration should be effected in the passage of the rays through the fluid.

4thly, That they should then impinge perpendicularly on the fourth surface, and be thence transmitted aplanatic to the focus.

Now it may be observed, that as far as relates to establishing the equations of condition, these will be the same as if the rays were refracted at each surface of the fluid lens from, and into, air; and as this method of considering the subject will throw some facilities into the investigation, it is adopted in what follows; but it is impossible not to believe that the practical effect must be very different in the two cases, seeing that in one the ray would have to sustain six considerable refractions, correcting each other by their positive and negative effects, while in the other there are only two inconsiderable refractions, equal to the difference.

Having thus settled the general principle of construction, the next step was to determine the dispersive ratio between the fluid and crown glass, these two media having never before to my knowledge been brought into optical combination. This was done by applying a crown lens to the dispersive instrument, described in the Phil. Trans. for 1827, p. 235; and as a proof of the accuracy of that method, it may be stated, that the calculations founded on the numbers thus obtained, answered most accurately to the focal length intended, and that the corrections for colour and spherical aberration were both as nearly perfect as is perhaps ever to be expected.

The numbers found as above, were

Index of sulphuret of carbon . . . $1+a' = 1{\cdot}6343$

——— crown glass . . . . $1+a = 1{\cdot}5396$

Dispersive ratio $1 : 3{\cdot}333$.

With these data and conditions we must now proceed to determine the radii of curvature of the lenses, by establishing and solving the following equations.

Let $nf =$ the focal length of the compound front lens.

$f =$ the part of that focus which falls beyond the fluid lens; and consequently, according to our condition,

$f =$ also the radius of curvature of the front surface of the first fluid shell.

$\frac{1}{x} =$ the sum of the reciprocals of the radii of curvature of the interior surfaces of the two shells, and consequently also of the fluid.

$y =$ the distance of the back surface of the second fluid shell from the focus; and consequently also, by our condition,

$y =$ the radius of the concave curvature of the back surface.

$1+a$ the index of crown glass.

$1+a'$ the index of the fluid; and

$a' - a = a''$.

Then it is obvious we must have

$$\frac{1}{f} + \left(\frac{1}{f} + \frac{1}{x}\right) a - \frac{1}{x} a' - \frac{1}{y} a = \frac{1}{y}$$

or
$$\frac{1+a}{f} - \frac{1+a}{y} = \frac{a''}{x}$$

whence
$$x = \frac{a'' f y}{(a+1)(y-f)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This equation has reference only to focus. We must now provide for achromatism, and for this purpose must determine the dispersive power of all the crown lens, considered as a simple lens, situate in the place of the fluid.

To effect this,

Let δ = the absolute dispersive power of crown glass.

$n\delta$ = the dispersive power of the front compound lens, as referred to its remaining focus at the place of the fluid lens.

f° = the focal length of the shells of the fluid lens.

Then
$$\left(\frac{1}{f}+\frac{1}{x}-\frac{1}{y}\right)a=\frac{1}{f^\circ}$$

or
$$f^\circ=\frac{fxy}{axy+af(y-x)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Now the dispersive power of these combined lenses, with that of the front lenses, reckoned from the place of the fluid, will be (denoting that power by δ'')

$$\delta''=\frac{f\delta+nf^\circ\delta}{f+f^\circ}$$

(See Journal of the Royal Institution, No. IV. p. 6.)

And to produce achromatism (remembering that $\frac{x}{a'}$ = focal length of the fluid, and $\frac{ff^\circ}{f+f^\circ}$ = the combined length of all the crown lenses,) we must have (calling δ''' the dispersive power of the fluid), to produce achromatism,

$$\frac{ff^\circ}{f+f^\circ}:\frac{x}{a'}::\delta'':\delta'''$$

Or substituting for δ'' its value as above found, and writing $m\delta$ for δ''', (m being the dispersive ratio between crown glass and the fluid,) we have

$$\frac{ff^\circ}{f+f^\circ}:\frac{x}{a}::\frac{f\delta+nf^\circ\delta}{f+f^\circ}:m\delta,$$

which when reduced gives

$$f^\circ=\frac{fx}{ma'f-nx}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

from which three equations, the values of x and y may be determined. That is, we first find

$$y=\frac{(ma'-a)(1+a)f-aa''f}{(ma'-a)(1+a)-(a+n)a''}$$

and then x is readily determined by means of equations (1) and (2).

In this expression for y; m, a, a', a'' are given quantities, but n is assumable at pleasure, within all practicable limits. In my former construction I found the rationality of the spectrum best preserved when n was taken $= 2$, and taking it so in the present instance with the proper value of the other quantities, viz. $a = \cdot 540$, $a' = \cdot 634$, $a'' = \cdot 094$, and $m = 3\cdot 333$, we have

$$y = \frac{1\cdot 573 \times 1\cdot 540 - \cdot 05076}{1\cdot 573 \times 1\cdot 540 - \cdot 23876} f = 1\cdot 086 f,$$

consequently $2\cdot 086 f = l =$ the whole length.

Hence any length whatever being assumed for a proposed telescope, or any value of f, all the other quantities may be immediately found. It is also quite indifferent what length is assumed, as all the other quantities will be proportional.

Assuming then a length $l = 150$ inches, we have

$$f = \frac{150}{2\cdot 086} = 71\cdot 9 \text{ inches}$$

$$y = 1\cdot 086 f = 78\cdot 1 \text{ inches, and}$$

$nf = 143\cdot 8$ inches, the combined focus of the front lens,

and equation (1) $x = 55\cdot 3$ inches, or

$\frac{1}{x} = \frac{1}{55\cdot 3} =$ the sum of the reciprocals of the radii of the interior surfaces of the fluid shells, or of the fluid itself, considered as negative.

If therefore in the particular case in question we denote the radii of the several curvatures of the fluid shells by r, r', r'', r''', we shall have $r = 71\cdot 9$, $r''' = -78\cdot 1$, and

$$\frac{1}{r'} + \frac{1}{r''} = \frac{1}{55\cdot 3}$$

and the whole focal length of the two combined front lenses $= 143\cdot 8$ inches.

Thus far then we have provided for focal length and achromatism, and have still r' and r'' undetermined, but which must now be found from these conditions; viz. so that

$$\frac{1}{r'} + \frac{1}{r''} = \frac{1}{55\cdot 3}$$

and that the spherical aberration at these surfaces may correct that of the front lenses, the radii of whose surfaces might also be left undetermined, we having at present only considered their focal length. It is proposed, however, that in this case these lenses shall be both equal plano-convex, which limits their radii, these being found one hundred and fifty-five inches.

The formula I employ for this determination is that given in Phil. Trans. for 1827, p. 247, observing only, that what is there denoted by d, which is the dispersive ratio, or the ratio of the focal length of the two lenses, must here be modified, in consequence of the lenses not being in contact, and the aberration being produced in the passage of the rays through the fluid, from glass to glass, that is, the effective index is on this account found by the proportion

$$1{\cdot}540 : 1{\cdot}634 :: 1 : 1{\cdot}061$$

we must therefore consider $a' = {\cdot}061$

and the effective focal length of the fluid $= - \frac{x}{a'} = - 906{\cdot}6$ inches $= f'$.

Referring now to the original formula above quoted, and putting C to denote the first factor, we shall find that the real equation is

$$C \times \frac{a' y'^2}{2 f' a' (q' + 1)} = \frac{p y^2}{2 f a}$$

which, in the common form, as $y' = y$, and $\frac{f}{f'} = d$, becomes

$$C \times \frac{d a}{q' + 1} = p$$

But in our case $y' = \frac{1}{2} y$; and our formula therefore is

$$C \times \frac{f a}{4 f' (q' + 1)}$$

consequent $d = \frac{f a}{4 f'} = {\cdot}0398$.

In the calculation for correcting aberration, we must therefore take p and d as above, $a = {\cdot}540$, $a' = {\cdot}061$, and then proceed exactly as in common cases, except that in this instance, as it is proposed to form the front lens of two plano-convex lenses, we must find the amount of aberration for such a combination, and then determine the ratio $r' : r''$ that shall produce in the fluid such amount of aberration as may correct that of the front lens; whereas, in the usual case, we first find the aberration of the concave lens, and adjust the front lens accordingly. By the latter method there is only required the solution of a quadratic equation, whereas, in reversing the operation, as here proposed, we arrive at a very intricate equation of difficult solution. Instead therefore of attempting a direct process by leaving the ratio $\frac{r'}{r''} = q$ indeterminate, it will be best to proceed by the method of position, which, though less elegant, is much more simple and expeditious. First, then, let us compute the amount of spherical aberration for parallel rays produced by two plano-convex lenses, the convex side of each being turned towards the object.

The aberration of a single convex lens, convex in front, whose focus is $f_{\prime}$ and diameter $2\,y$, is

$$\text{aberration} = \frac{a^4 + 2\,a^3 + 1}{2\,a\,(a^3 + 2\,a^2 + a)} \times \frac{y^2}{f_{\prime}}$$

which, when $a = {\cdot}540$, becomes $\frac{1{\cdot}012\,y^2}{f_{\prime}}$

and since the focus of the combined lenses is 143·8 inches, that of this single lens is $f_{\prime} = 287{\cdot}8$ inches, and $\frac{287{\cdot}8}{{\cdot}540} = 155$ inches = radius of the front surface.

At the second lens, therefore, the rays are converging to a distance $d = -\,287{\cdot}6$; and this lens being equal to the former, its radius $r = 155{\cdot}3$, r' being infinite, and it remains to determine its aberration.

Calling $\frac{d}{r} = \frac{-\,287{\cdot}6}{155{\cdot}3} = m = -\,1{\cdot}85$, and

$$\frac{(a+1)\,d\,r}{a\,d - r} = d' = 1{\cdot}424\,r$$

$$\frac{a}{a+1} = b = {\cdot}35$$

the formula for the aberration (Phil. Trans. for 1827, p. 243,) becomes

$$\left.\begin{array}{l} \frac{a\,(m+1)^2}{(a\,m-1)^2} \times \frac{m+a+2}{(a+1)\,m} \times \frac{1-b}{2\,a} \\ + \frac{b}{1} \times \frac{2-b}{2\,(b-1) \times 1{\cdot}424\,r} \end{array}\right\} \times \frac{y^2}{f_{\prime}}$$

which in numbers gives

$$\text{aberration} = \frac{{\cdot}5629\,y^2}{f_{\prime}}$$

of the second lens only. We have already found in the first lens, aberration

$= \frac{1{\cdot}012\, y^2}{f_{\prime}}$; but this is reduced by the second lens to one fourth its amount, or to $\frac{253\, y^2}{f_{\prime}}$, whence the aberration of the combined lenses is $\frac{{\cdot}8159\, y^2}{f_{\prime}}$ or $\frac{{\cdot}4079\, y^2}{nf}$, nf denoting the compound focus $= \frac{1}{2} f_{\prime}$ according to our first notation.

And it remains to find such a ratio of $r' : r''$ or such a value of $\frac{r''}{r'} = q'$ as shall produce the aberration in question.

The formulæ for the calculation of this aberration with any given value of q' already referred to in the Phil. Trans. are,

$$* \, c = \frac{f''}{r'} \qquad c' = \frac{(a'+1) f'' q'}{a' f'' - r''} \qquad b = \frac{a}{a+1}, \text{ and then}$$

$$\left. \begin{array}{l} \frac{(c+q')^2}{(a'c-q')^2} \times \frac{c+(a'+2)\,q'}{c\,(a'c'+a'+1)^2} \\ + \frac{(c'+1)^2}{(bc'+1)^2} \times \frac{(c'+2-b)\,q'}{c'} \end{array} \right\} \frac{a\,d}{q'+1} = p$$

in this equation $p = {\cdot}4079$, $a = {\cdot}540$, $a' = {\cdot}061$, $f'' = y = 78$, and $d = {\cdot}0398$, $\frac{r''}{r'} = q'$ being at present undetermined, but which is now to be assumed so as to give an approximate value to p.

A few simple trials will show that the value of q' must be about -3. Assuming then $q' = -3$, we have

$$r'' = f' a' (q'+1) = 110{\cdot}6$$

$$r' = f' a' \frac{(q'+1)}{q'} = -37$$

and as we already know $r = 72$, and $r''' = -78$ (taking only their nearest integral values,) we find by substitution in the preceding formula, $p = {\cdot}4211$ instead of $p = {\cdot}4079$.

Take therefore $q = -3\frac{1}{3}$, then

$$r'' = 129, \text{ and } r' = -38{\cdot}7$$

which, by substitution, gives $p = {\cdot}3930$.

It is clear therefore that the value of q lies between the limits -3 and $-3{\cdot}333$,

* It will be observed that r' is here the same as r'' in the original formula, and r'' the same as r'''.

and it will be observed that these values of p, vary principally as the factor $\frac{q'}{q'+1}$ varies; and treating the subject upon the common arithmetical principle of position, we find $\frac{q'}{q'+1} = 1{\cdot}4565$, or $q' = -3{\cdot}2$ very nearly.

This value of q gives

$$r'' = f' a' (q' + 1) = 121{\cdot}5 \text{ inches}$$

$$r' = f' a' \frac{(q' + 1)}{q'} = -38 \text{ inches.}$$

We have now therefore all the numbers answering to a total length of 150 inches, viz.

l = 150 inches whole length,
r = 72 inches first surface of fluid shell,
r' = 38 inches second surface,
r'' = $-$ 121·5 = third surface,
r''' = $-$ 78 inches fourth surface,
R = 155·3 inches convex surface of front lens,
Δ = 72 inches distance of the lenses.

And it is only necessary for any proposed lengths to use these proportions.

On consulting with Mr. Dollond, I found that to have confined myself to eight feet length, two or three new tools would have been necessary, but by taking $l = 105$ inches, he had only occasion for one. I therefore adopted that length; for although Mr. Dollond was very desirous of working out my numbers to any accuracy I wished, I had no desire to urge this point to an unnecessary degree of refinement, particularly as the nature of the construction always admits of producing chromatic correction to any degree of precision.

I have already stated that the lenses worked according to these proportions, when placed in their respective cells, agreed in every respect with the computed results, as well in focus as in chromatic and spherical correction.

These remarks, it will be observed, apply to the arrangement of the fluid lens according to the first principle, in which the corrections are all made in the passage of the light through the fluid, and by the fluid only. By reversing this lens we have a telescope on the second principle, in which the correcting power of the fluid for spherical aberration is a minimum, or at least very small in comparison with that of the glass in which it is inclosed; and on trial it was found, I must admit contrary to my expectation, that the performance of the telescope was better with the lens in this position than in the former; the difference however is but little, and this little has reference only to figure; I therefore am rather inclined to attribute this preference to some accidental better adaptation of the glasses to each other when in this position than to any actual theoretical advantage. In both cases the spherical aberration is equally well corrected, there being no perceptible change of focus, whether we employ the whole aperture or the central four inches, or a simple ring of an inch of light round the margin of the lens. In each of these three cases I have repeatedly seen the small star in Rigel without the slightest change of focus. For the rest it is not my intention to offer any remarks on the performance of the telescope, as I conceive this will be best reported upon by the committee, after they have submitted it to such tests as may be judged most proper for determining its defining and penetrating power.

It only therefore remains for me to state, that I feel much indebted to Mr. Dollond for the readiness with which he has complied with all my suggestions, and for the accuracy with which he has executed every part of the instrument.

P.S. Since this paper was written, Mr. Dollond has reworked one of the surfaces, and there is now no question that the preference in the performance is decidedly with the fluid lens in its first position.